Public Announcement Logic with Misinterpretations

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Misinterpretations: Ambiguity

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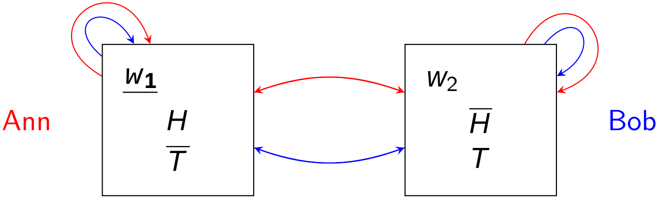
After observing that the coin lands on its heads, Ann believes that H is true, while Bob believes that T is true.

The Unfamiliar Coin: via an Action model



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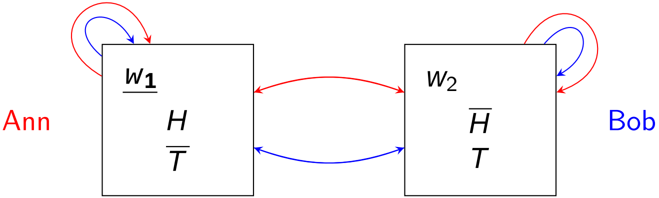

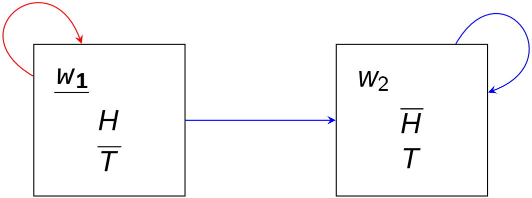

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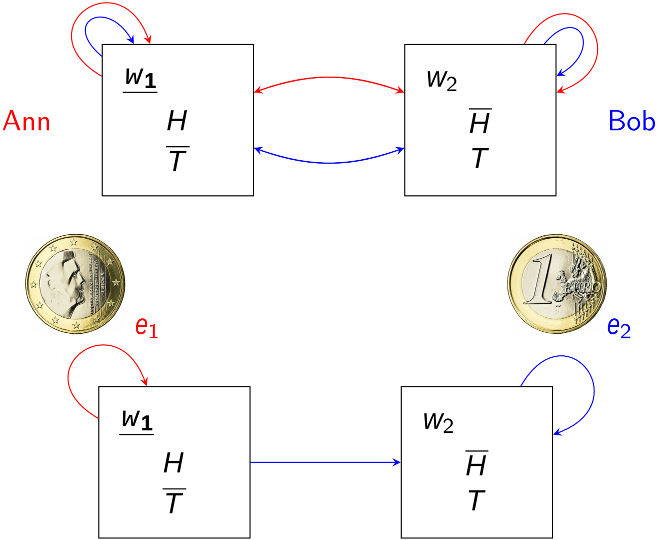
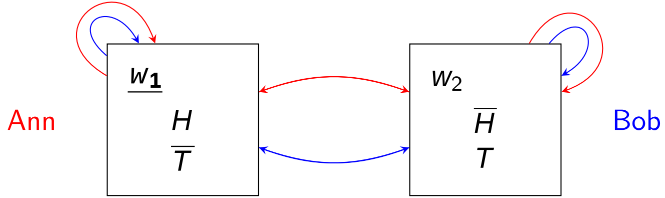


The Unfamiliar Coin: Our Approach



$[H]$

- A syntactical approach to model misinterpretations.
- Agent-relative interpretation functions $_i$:

$$_{Ann}(H) = H \quad \text{but} \quad _{Bob}(H) = T$$

Formal Definitions: Syntax

- At: a CI set of atomic propositions
- Lit $= \mathrm{At} \cup \{\overline{p} \mid p \in \mathrm{At}\}$
- The propositional language L_0 for At:

$$::= p \mid \overline{p} \mid \bot \mid \top \mid (\quad \wedge \quad) \mid (\quad \vee \quad)$$

where $p, \overline{p} \in$ Lit.

Formal De nitions: Agent-relative interpretation $_i$

Given a set of agents G, for all $i \in G$,

$$_i : Lit \to L \cup \{ \quad I \}$$

with $I \notin At$ being a special symbol for `ignored', and

$_i(p) = \ p$ i $\quad _i(\overline{p}) = \ \overline{p}$ and $_i(p) = I$ i $\quad _i(\overline{p}) = I$.

Formal De nitions: Agent-relative interpretation i

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i (p) = p i i (p̄) = p̄ and i (p) = I i i (p̄) = I.

Lifting i to all formulas of L_0:

~i (p) = i (p)

~i (p̄) = i (p̄)

~i (>) = i (>)

~i (?) = i (?)

~i (' ^) = 8 > < > : ~i (') if ~i ( ) = I ; ~i ( ) if ~i (') = I ; ~i (') ^ ~i ( ) otherwise

~i (' _) = 8 > < > : ~i (') if ~i ( ) = I ; ~i ( ) if ~i (') = I ; ~i (') _ ~i ( ) otherwise

Formal Definitions: Model & Announcement Dynamics

Model

$$\langle W, (R_i)_{i \in G}, (\leq_i)_{i \in G}, V \rangle$$

where R_i is serial, transitive and Euclidean.

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$M, w \models [\ \ ]_i \varphi$ iff if $M, w \models \ \ $, then $M^{[\ \ ]_i}, v \models \varphi$

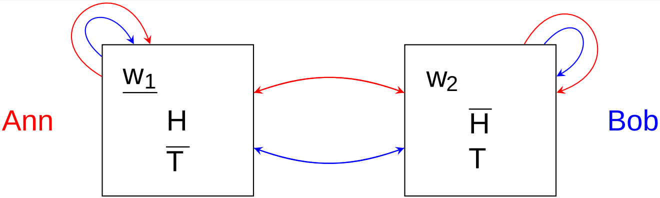
—Update on R_i

If $\ _i(\ \ ) \neq I$, then:

$R_i^{[\ \ ]_i} = R \cap \{(w, v) \mid (M, w \models \ \ _i(\ \ ) \text{ or } M, w \models \ \ ) \text{ and } M, v \models \ \ _i(\ \ )\}$

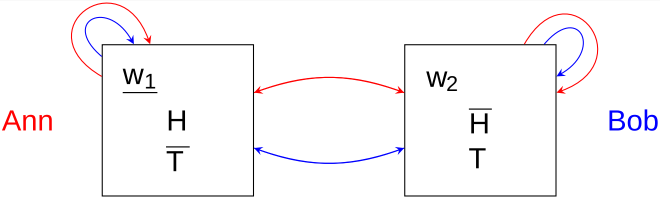
otherwise, $R_i^{[\ \ ]_i} = R$.

The Unfamiliar Coin



$_{\text{Ann}}(H) = H$ $[H]$ $_{\text{Bob}}(H) = T$

The Unfamiliar Coin



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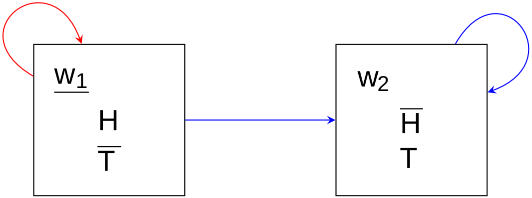

Misinterpretations: Lack of Inquisitive Interest

[From A. Baltag, R. Boddy, and S. Smets. Group knowledge in interrogative epistemology, 2018. Modi ed]

Ann the logician and Bob the philosopher are the two member of a hiring committee for an academic position. They are looking at the writing samples respectively written by two candidates, Chloe and Dan.

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Chloe is the better logician, and Dan is the better philosopher.

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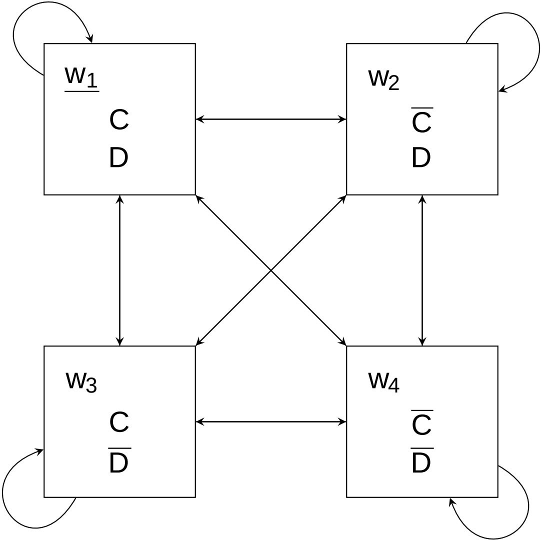
Chloe is the better logician, and Dan is the better philosopher.

On the one hand, Ann who is not interested in the candidates' competen in philosophy comes to believe that Chloe is the better candidate. On th other hand, Bob who is not interested in the candidates' competency in logic comes to believe that Dan is the better candidate.

The Curse of Committee Modi ed



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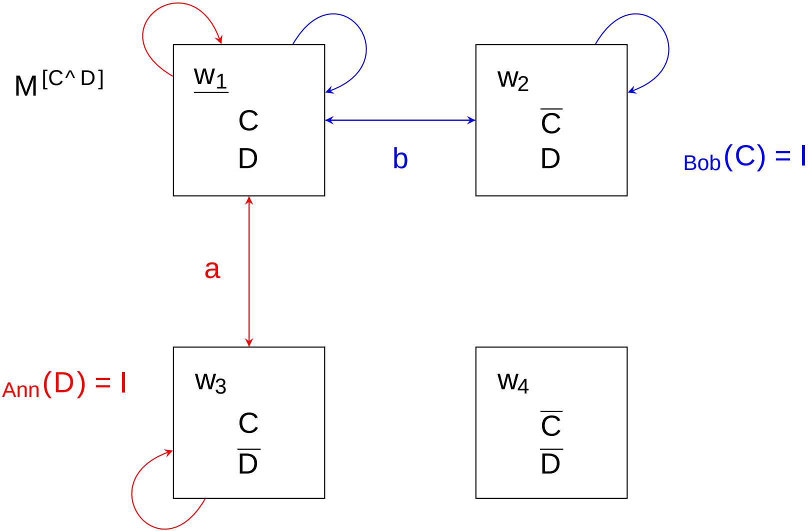


Formal Definitions: Correction Dynamics

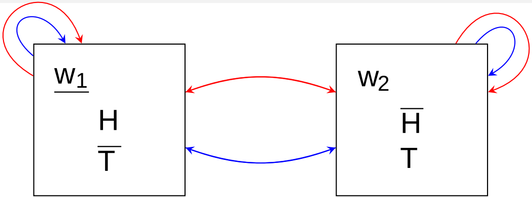
$M, w \models [\;!]_i \varphi$ iff if $M, w \models$, then $M^{[\;!]_i}, v \models \varphi$

—Update on $_i$

Define $^{[\;!]_i}_i : Lit \to L$ as follows:

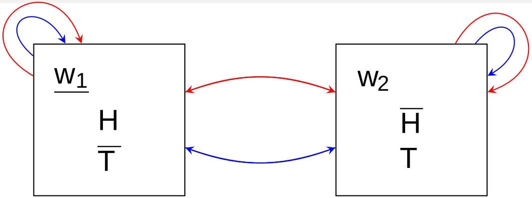
- for all $p \in At$, if p or $\overline{p}$ appears in , then $^{[\;!]_i}_i(p) = p$ and $^{[\;!]_i}_i(\overline{p}) = \overline{p}$
- $^{[\;!]_i}_i(x) = x$ otherwise.

The Unfamiliar Coin

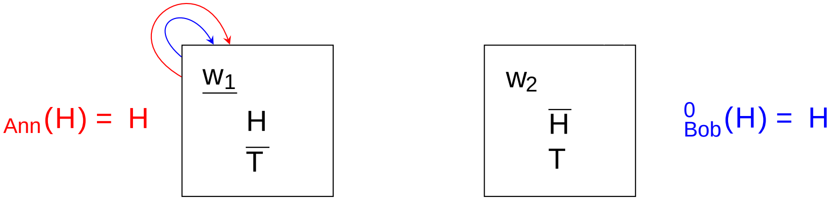


[H!][H]

The Unfamiliar Coin



[H!][H]



Correction & Belief Revision

[']

The announcement may not be successful.

[' !][']

The announcement is successful.

Correction & Belief Revision

[']

The announcement may not be successful.

[' !][']

The announcement is successful.

['][' !]

Would the correction of misinterpretation also correct one's beliefs?

Correction & Belief Revision

Ann and Bob are Italian, and most Italians call Beijing Pechino. In 2008, Bob was reading the news and learned that Beijing was hosting the Olympics. Not knowing that Beijing is Pechino, Bob believed that some small town in China was hosting the Olympics.

Correction & Belief Revision

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Ann, who realized Bob's misunderstanding, corrected him by pointing ou Beijing is Pechino. Afterwards, Bob started believing that Beijing, which is the capital of China, was hosting the Olympics.

What happened?

$$\text{NOT} \quad {}_{\text{Bob}}(\text{Beijing}) = \text{a small town} \,\&\, {}^{0}_{\text{Bob}}(\text{Beijing}) = \text{Pechino}$$

What happened?

NOT $_{\text{Bob}}$(Beijing) = a small town & $^{0}_{\text{Bob}}$(Beijing) = Pechino

RATHER Bob misunderstood the properties of an object `Beijing'.

The correction replaced the incorrect properties of `Beijing' with thos of `Pechino'.

Future Direction

Current: agents misinterpret formulas.

Future Direction

Current: agents misinterpret formulas.

Future: agents misinterpret objects and/or their properties.

Related Works

Baltag, A., Boddy, R., Smets, S.: Group knowledge in interrogative epistemology. In: Ditmarsch, H.V., Sandu, G. (eds.) Jaakko Hintikka on Knowledge and Game Theoretical Semantics. Springer (2018)

Bjorndahl, A., Özgün, A.: Uncertainty about evidence. In: Moss, L.S. (ed.) Proceedings Seventeenth Conference on Theoretical Aspects of Rationality and Knowledge, TARK 2019, Toulouse, France, 17-19 July 2019. EPTCS, vol. 297, pp. 68{81 (2019).

Halpern, J.Y., Kets, W.: Ambiguous language and di erences in beliefs (2012).

Heifetz, A., Meier, M., Schipper, B.: Interactive unawareness. Journal of Economic Theory 130(1), 78{94 (2006).

van Ditmarsch, H., van der Hoek, W., Kooi, B.: Dynamic epistemic logic with assignment. In: AAMAS '05: Proceedings of the fourth international joint conference on Autonomous agents and multiagent systems. pp. 141 { 148 (2005)

Thank you!

Formal Definitions: Syntax

- G: a set of agents
- At: a CI set of atomic propositions
- Lit $=$ At *[f$\overline{p}$ j p 2* At*g*
- The propositional language L_0 for At:

$$::= \; p \; j \; \overline{p} \; j \; ? \; j \; > \; j \; (\quad \wedge \quad) \; j \; (\quad \_ \quad)$$

where *p;$\overline{p}$ 2* Lit.

- The language L:

$$' \quad ::= \quad j \; : ' \; j \; (' \wedge ') \; j \; (' \_ ') \; j \; B_i \; ' \; j \; B_i \; ' \; j \; [\quad]_i \; ' \; j \; [\quad !]_i \; ' \; j \; (\quad ._i \; ')$$

where *i 2* G and *2* Lit.

Formal Definitions: Model & Semantics

$$\langle W, (R_i)_{i \in G}, (\ \ _i)_{i \in G}, V \rangle$$

where R_i is serial, transitive and Euclidean.

- $M, w \models p$ iff $w \in V(p)$
- $M, w \models \overline{p}$ iff $w \notin V(p)$
- $M, w \not\models \bot$
- $M, w \models \top$
- $M, w \models \neg \varphi$ iff $M, w \not\models \varphi$
- $M, w \models \varphi \vee \ $ iff $M, w \models \varphi$ or $M, w \models$
- $M, w \models \varphi \wedge \ $ iff $M, w \models \varphi$ and $M, w \models$
- $M, w \models B_i \varphi$ iff for all $v \in W$, if $w R_i v$, then $M, v \models \varphi$
- $M, w \models \ ._i\ \varphi$ iff $\ _i(\ ) = \varphi$

Formal Definitions: Explicit Belief

- $\Lambda_i = f \, ' \, j$ for some $\; 2 \, L_0; \; {}_i(\;) = \, ' g$ [i's awareness set]
- $W^{?}_{i} = fw \, j \, 9 \, '$ s.t. $\, '; : ' \; 2 \; \Lambda_i^w g$ [i's -impossible worlds]
  where $\Lambda_i^w = f \, ' \, j \, 9 \;$ s.t. $M; w \; j\!= \;$ and $\; {}_i(\;) = \, ' g$
- $W_{\; i} = W n W^{?}_{i}$ [i's -possible world]
- $R_i \; = R_i \; \backslash \; (W_{\; i} \quad W_{\; i})$ [Explicit doxastic relation]
- $M; w \; j\!= B_i \, '$ iff $9 \;$ s.t. $\; {}_i(\;) = \, '$ and $M; v \; j\!= \;$ for all $v \; 2 \; R_i \, (w)$ [i's explicit belief]