Public Announcement Logic with Misinterpretations

Masayuki Tashiro, Eric Pacuit, Ilaria Canavotto

Department of Philosophy
University of Maryland, College Park

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Misinterpretations: Ambiguity

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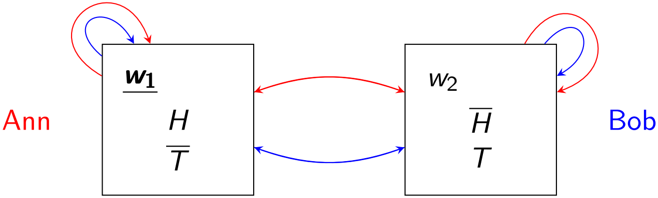
After observing that the coin lands on its heads, Ann believes that H is true, while Bob believes that T is true.

The Unfamiliar Coin: via an Action model



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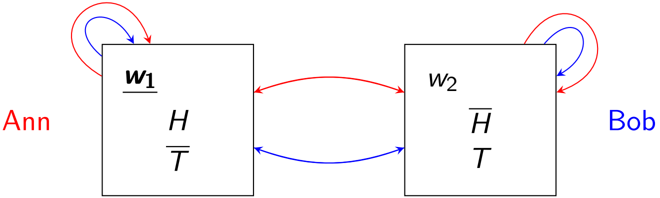

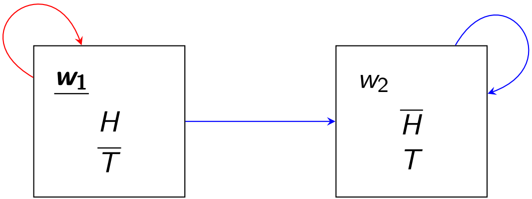

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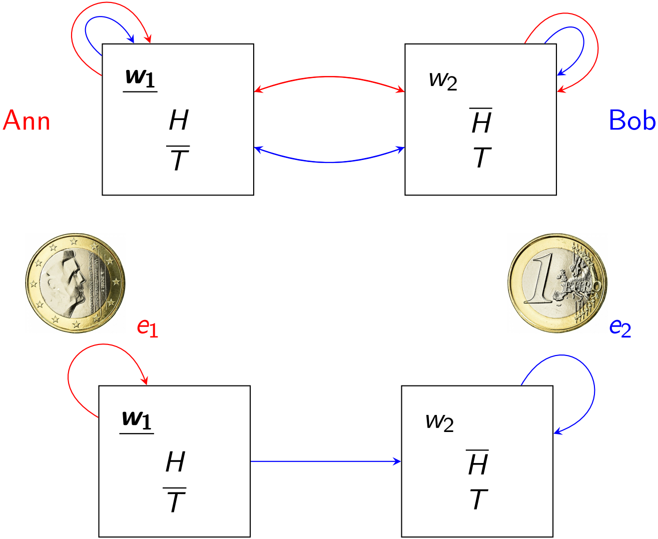
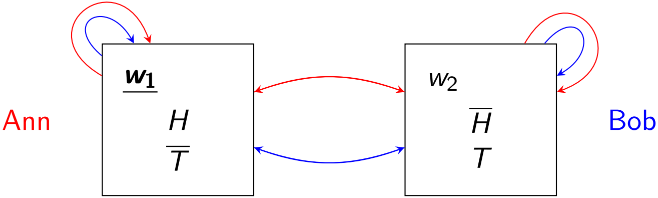


The Unfamiliar Coin: Our Approach



$[H]$

- A syntactical approach to model misinterpretations.
- Agent-relative interpretation functions λ_i:

$$\lambda_{Ann}(H) = H \quad \text{but} \quad \lambda_{Bob}(H) = T$$

Formal Definitions: Syntax

- At: a CI set of atomic propositions
- $\mathrm{Lit} = \mathrm{At} \cup \{\overline{p} \mid p \in \mathrm{At}\}$
- The propositional language $\mathcal{L}_0$ for At:

$$\alpha ::= \; p \mid \overline{p} \mid \bot \mid \top \mid (\alpha \wedge \alpha) \mid (\alpha \vee \alpha)$$

where $p, \overline{p} \in \mathrm{Lit}$.

Formal Definitions: Agent-relative interpretation λ_i

- Given a set of agents G, for all $i \in$ G,

$$\lambda_i : \mathsf{Lit} \to \mathcal{L} \cup \{\mathsf{I}\}$$

with $\mathsf{I} \notin \mathsf{At}$ being a special symbol for 'ignored', and
$\lambda_i(p) = p$ iff $\lambda_i(\overline{p}) = \overline{p}$ and $\lambda_i(p) = \mathsf{I}$ iff $\lambda_i(\overline{p}) = \mathsf{I}$.

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- Lifting λ_i to all formulas of $\mathcal{L}_0$:

$$\tilde{\lambda}_i(p) = \lambda_i(p) \qquad \tilde{\lambda}_i(\varphi \wedge \psi) = \begin{cases} \tilde{\lambda}_i(\varphi) & \text{if } \tilde{\lambda}_i(\psi) = \mathsf{I} \\ \tilde{\lambda}_i(\psi) & \text{if } \tilde{\lambda}_i(\varphi) = \mathsf{I} \\ \tilde{\lambda}_i(\varphi) \wedge \tilde{\lambda}_i(\psi) & \text{otherwise} \end{cases}$$

$$\tilde{\lambda}_i(\overline{p}) = \lambda_i(\overline{p})$$

$$\tilde{\lambda}_i(\top) = \lambda_i(\top) \qquad \tilde{\lambda}_i(\varphi \vee \psi) = \begin{cases} \tilde{\lambda}_i(\varphi) & \text{if } \tilde{\lambda}_i(\psi) = \mathsf{I} \\ \tilde{\lambda}_i(\psi) & \text{if } \tilde{\lambda}_i(\varphi) = \mathsf{I} \\ \tilde{\lambda}_i(\varphi) \vee \tilde{\lambda}_i(\psi) & \text{otherwise} \end{cases}$$

$$\tilde{\lambda}_i(\bot) = \lambda_i(\bot)$$

Formal Definitions: Model & Announcement Dynamics

- Model

$$\langle W, (R_i)_{i\in \mathsf{G}}, (\lambda_i)_{i\in \mathsf{G}}, V\rangle$$

where R_i is serial, transitive and Euclidean.

Formal Definitions: Model & Announcement Dynamics

- Model

$$\langle W, (R_i)_{i \in \mathsf{G}}, (\lambda_i)_{i \in \mathsf{G}}, V \rangle$$

where R_i is serial, transitive and Euclidean.

- $\mathcal{M}, w \models [\alpha]_i \varphi$ iff if $\mathcal{M}, w \models \alpha$, then $\mathcal{M}^{[\alpha]_i}, v \models \varphi$

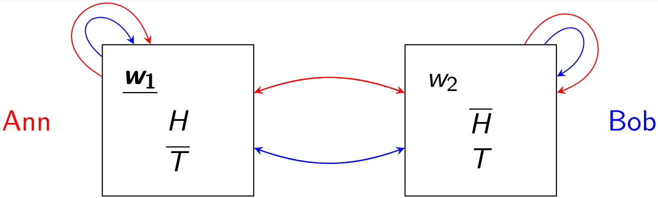
—Update on R_i

If $\lambda_i(\alpha) \neq \mathsf{I}$, then:

$R_i^{[\alpha]_i} = R \cap \{(w, v) \mid (M, w \models \lambda_i(\alpha) \text{ or } M, w \models \alpha) \text{ and } M, v \models \lambda_i(\alpha)\}$

otherwise, $R_i^{[\alpha]_i} = R$.

The Unfamiliar Coin



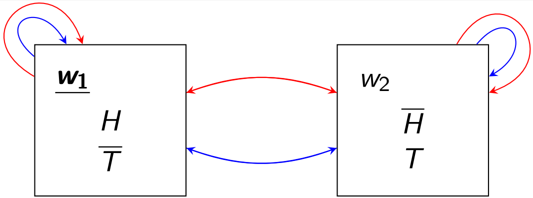
$\lambda_{Ann}(H) = H$

$[H]$

$\lambda_{Bob}(H) = T$

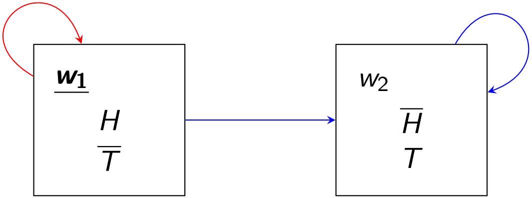
The Unfamiliar Coin

Ann



Bob

$\lambda_{Ann}(H) = H$ $[H]$ $\lambda_{Bob}(H) = T$



Misinterpretations: Lack of Inquisitive Interest

[From A. Baltag, R. Boddy, and S. Smets. Group knowledge in interrogative epistemology, 2018. *Modified*]

Ann the logician and Bob the philosopher are the two member of a hiring committee for an academic position. They are looking at the writing samples respectively written by two candidates, Chloe and Dan.

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The writing samples (objectively) indicate that:

Chloe is the better logician, and Dan is the better philosopher.

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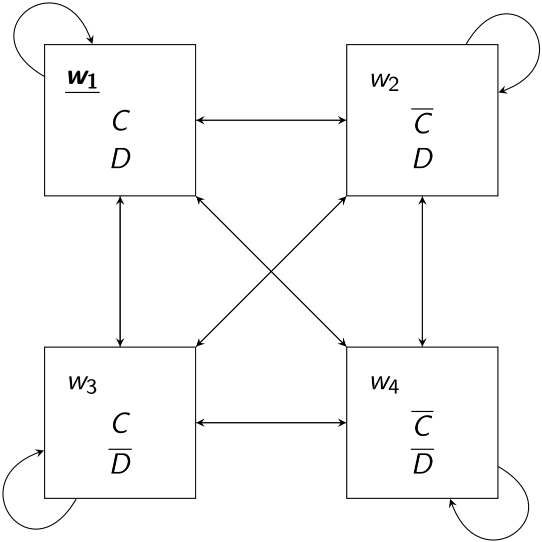
The writing samples (objectively) indicate that:

Chloe is the better logician, and Dan is the better philosopher.

On the one hand, Ann who is not interested in the candidates' competency in philosophy comes to believe that Chloe is the better candidate. On the other hand, Bob who is not interested in the candidates' competency in logic comes to believe that Dan is the better candidate.

The Curse of Committee Modified



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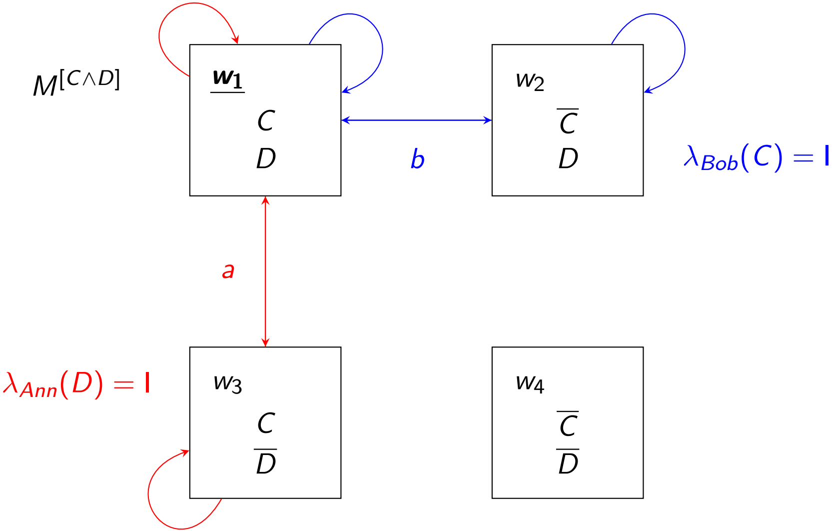


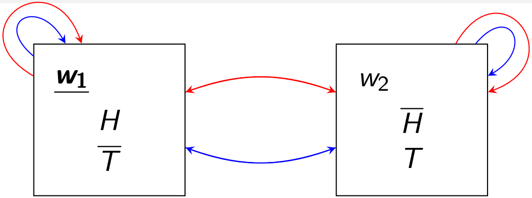
Formal Definitions: Correction Dynamics

- $\mathcal{M}, w \models [\alpha!]_i\varphi$ iff if $\mathcal{M}, w \models \alpha$, then $\mathcal{M}^{[\alpha!]_i}, v \models \varphi$

  —Update on λ_i

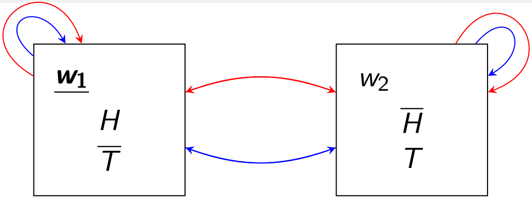
  Define $\lambda_i^{[\alpha!]_i}$: Lit $\to \mathcal{L}$ as follows:
  - for all $p \in$ At, if p or $\overline{p}$ appears in α, then
  $$\lambda_i^{[\alpha!]_i}(p) = p \text{ and } \lambda_i^{[\alpha!]_i}(\overline{p}) = \overline{p}$$
  - $\lambda_i^{[\alpha!]_i}(x) = x$ otherwise.

The Unfamiliar Coin



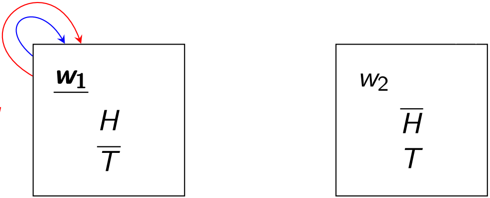
$[H!][H]$

The Unfamiliar Coin



$[H!][H]$

$\lambda_{Ann}(H) = H$



$\lambda'_{Bob}(H) = H$

Correction & Belief Revision

- $[\varphi]$

  The announcement may not be successful.

- $[\varphi!][\varphi]$

  The announcement is successful.

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- $[\varphi][\varphi!]$

  Would the correction of misinterpretation also correct one's beliefs?

Correction & Belief Revision

Ann and Bob are Italian, and most Italians call Beijing *Pechino*. In 2008, Bob was reading the news and learned that Beijing was hosting the Olympics. Not knowing that Beijing is *Pechino*, Bob believed that some small town in China was hosting the Olympics.

Correction & Belief Revision

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Ann, who realized Bob's misunderstanding, corrected him by pointing out Beijing is *Pechino*. Afterwards, Bob started believing that Beijing, which is the capital of China, was hosting the Olympics.

What happened?

- **NOT** $\lambda_{Bob}(Beijing) = a\ small\ town\ \&\ \lambda'_{Bob}(Beijing) = Pechino$

What happened?

- **NOT** $\lambda_{Bob}(Beijing) = a\ small\ town\ \&\ \lambda'_{Bob}(Beijing) = Pechino$
- **RATHER** Bob misunderstood the properties of an object 'Beijing'.
- The correction replaced the incorrect properties of 'Beijing' with those of 'Pechino'.

Future Direction

- **Current**: agents misinterpret formulas.

Future Direction

- **Current**: agents misinterpret formulas.
- **Future**: agents misinterpret objects and/or their properties.

Related Works

- Baltag, A., Boddy, R., Smets, S.: Group knowledge in interrogative epistemology. In: Ditmarsch, H.V., Sandu, G. (eds.) Jaakko Hintikka on Knowledge and Game Theoretical Semantics. Springer (2018)
- Bjorndahl, A., Özgün, A.: Uncertainty about evidence. In: Moss, L.S. (ed.) Proceedings Seventeenth Conference on Theoretical Aspects of Rationality and Knowledge, TARK 2019, Toulouse, France, 17-19 July 2019. EPTCS, vol. 297, pp. 68–81 (2019).
- Halpern, J.Y., Kets, W.: Ambiguous language and differences in beliefs (2012).
- Heifetz, A., Meier, M., Schipper, B.: Interactive unawareness. Journal of Economic Theory 130(1), 78–94 (2006).
- van Ditmarsch, H., van der Hoek, W., Kooi, B.: Dynamic epistemic logic with assignment. In: AAMAS '05: Proceedings of the fourth international joint conference on Autonomous agents and multiagent systems. pp. 141 – 148 (2005).

Thank you!

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where $p, \overline{p} \in \mathrm{Lit}$.

- The language $\mathcal{L}$:

$$\varphi \; ::= \; \alpha \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid (\varphi \vee \varphi) \mid B_i\varphi \mid B_i^{\triangleright}\varphi \mid [\alpha]_i\varphi \mid [\alpha!]_i\varphi \mid (\alpha \triangleright_i \varphi)$$

where $i \in \mathrm{G}$ and $\alpha \in \mathrm{Lit}$.

Formal Definitions: Model & Semantics

$$\langle W, (R_i)_{i \in \mathsf{G}}, (\lambda_i)_{i \in \mathsf{G}}, V \rangle$$

where R_i is serial, transitive and Euclidean.

- $\mathcal{M}, w \models p$ iff $w \in V(p)$
- $\mathcal{M}, w \models \overline{p}$ iff $w \notin V(p)$
- $\mathcal{M}, w \not\models \bot$
- $\mathcal{M}, w \models \top$
- $\mathcal{M}, w \models \neg\varphi$ iff $\mathcal{M}, w \not\models \varphi$
- $\mathcal{M}, w \models \varphi \vee \psi$ iff $\mathcal{M}, w \models \varphi$ or $\mathcal{M}, w \models \psi$
- $\mathcal{M}, w \models \varphi \wedge \psi$ iff $\mathcal{M}, w \models \varphi$ and $\mathcal{M}, w \models \psi$
- $\mathcal{M}, w \models B_i\varphi$ iff for all $v \in W$, if $w\ R_i\ v$, then $\mathcal{M}, v \models \varphi$
- $\mathcal{M}, w \models \alpha \triangleright_i \varphi$ iff $\lambda_i(\alpha) = \varphi$

Formal Definitions: Explicit Belief

- $\Lambda_i = \{\varphi \mid \text{for some } \psi \in \mathcal{L}_0, \lambda_i(\psi) = \varphi\}$ [i's awareness set]
- $W^{\perp}_{\lambda_i} = \{w \mid \exists\varphi \text{ s.t. } \varphi, \neg\varphi \in \Lambda^w_i\}$ [i's λ-impossible worlds]
  where $\Lambda^w_i = \{\varphi \mid \exists\psi \text{ s.t. } M, w \models \psi \text{ and } \lambda_i(\psi) = \varphi\}$
- $W_{\lambda_i} = W \backslash W^{\perp}_{\lambda_i}$ [i's λ-possible world]
- $R^{\lambda}_i = R_i \cap (W_{\lambda_i} \times W_{\lambda_i})$ [Explicit doxastic relation]
- $M, w \models B^{\triangleright}_i\varphi$ iff $\exists\alpha$ s.t. $\lambda_i(\alpha) = \varphi$ and $M, v \models \alpha$ for all $v \in R^{\lambda}_i(w)$ [i's explicit belief]